Sec 1.1 Functions and Function Notation

Function – a rule that takes certain numbers as input and assigns to each input exactly one output number.

Ex.: f(6) = 10 6 is the input and the output is 10

A function can be represented in 4 ways:

1. Words – a description of what is occurring
2. Table – chart with independent (inputs) and dependent (outputs) variables
3. Graph – the pair of values represented in the coordinate plane
4. Formula – an equation that represents the overall spectrum of the graph

Mathematical Model – when we use a function to describe an actual situation, this can be used to make predictions about the function

Function Notation: to indicate that a quantity Q is a function of a quantity t, we abbreviate Q = f(t). This means that by applying the rule f to the input t, we get the output of Q, ie. Dependent = f(independent).

Ex. The number of gallons of paint needed to paint a house depends on the size of the house and a gallon typically covers 250 square feet. Thus, the number of gallons of paint, n, can be found by n = f(A).

a. Find a formula for f. $f(A) = \frac{A}{250}$
b. Explain in words what the statement f(10,000) = 40 means.

An area of 10,000 square feet requires 40 gallons of paint.

Ex. If T = .25 R + 40, then T = f(R). If R is 10, ~~right~~ write in function notation and then find T. Create a formula for the input and output of the function.

$f(10) = .25(10) + 40$
$= 2.5 + 40$
$f(10) = 42.5$

$T = 42.5$

$T = .25R + 40$
$4(T - 40) = .25R \cdot 4$
$4T - 160 = R$ or $R = f(T) = 4T - 160$

Finding Values of a Function – substitute f(x) in for every x and simplify

Ex. For the function $f(x) = 2x^2 + 3x$, evaluate:

a. f(2)
$2(2)^2 + 3(2)$
$2 \cdot 4 + 6$
$8 + 6$
14

b. f(x) + f(3)
$2x^2 + 3x + 2(3)^2 + 3(3)$
$2x^2 + 3x + 2(9) + 9$
$2x^2 + 3x + 18 + 9$
$2x^2 + 3x + 27$

c. f(-x)
$2(-x)^2 + 3(-x)$
$2x^2 + -3x$
$2x^2 - 3x$

d. –f(x)
$-(2x^2 + 3x)$
$-2x^2 - 3x$

e. f(x – 2)
$2(x-2)^2 + 3(x-2)$
$2(x^2 - 4x + 4) + 3x - 6$
$2x^2 - 8x + 8 + 3x - 6$
$2x^2 - 5x + 2$

f. f(x + h) – f(x)
$2(x+h)^2 + 3(x+h) - (2x^2 + 3x)$
$2(x^2 + 2xh + h^2) + 3x + 3h - 2x^2 - 3x$
$2x^2 + 4xh + 2h^2 + 3h - 2x^2$
$2h^2 + 4xh + 3h$ or
$h(2h + 4x + 3)$

Function – from x to y is a relation that associates each X to exactly one Y

a. Domain – the set X, what can be input
b. Range – the set of all images of the domain, a subset of Y

Determining a Function: check to see if every x corresponds to only one y (some y values may be used more than once or not used at all)

Ex. Is this a function?

A.
Katy → June 20
Jake → April 10
Cedric → Feb. 13
Henry → Feb. 13

Function

B.
Dave → 555-0273
Don → 382-9382
April → 382-8291
Sarah → 829-5692
Sarah → 918-8165

Not a Function

C. {(1, 4) (2, 4) (3, 5) (6, 10)}

Function

D. {(-3, 9) (-2, 4) (0, 0) (1, 1) (-3, 8)}

Not a Function

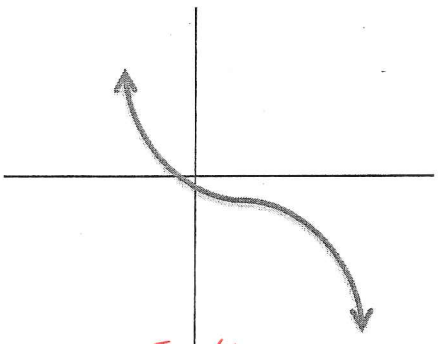
E. {(1, 4) (2, 5) (3, 6) (4, 7)}

Function

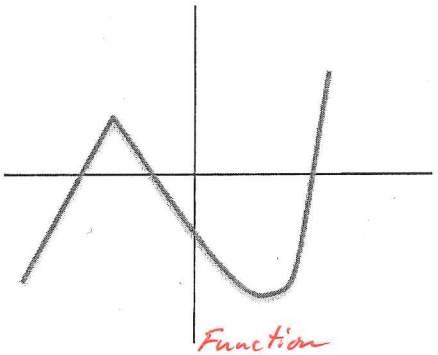
To determine if it is a function:

a. Solve for y.
b. Either make a table of values or graph then check table for values that occur twice (ie the same x appears twice for different y values)
c. Vertical Line Test – if there is a vertical line that intersects the graph at more than one point, it is not a function

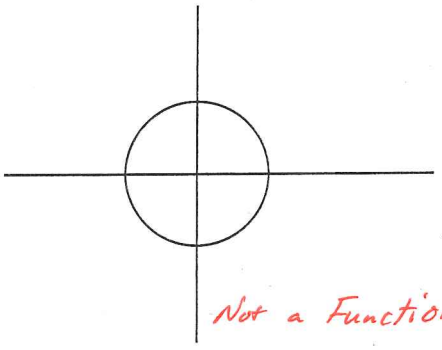
Ex. Which of the following are functions?



Function



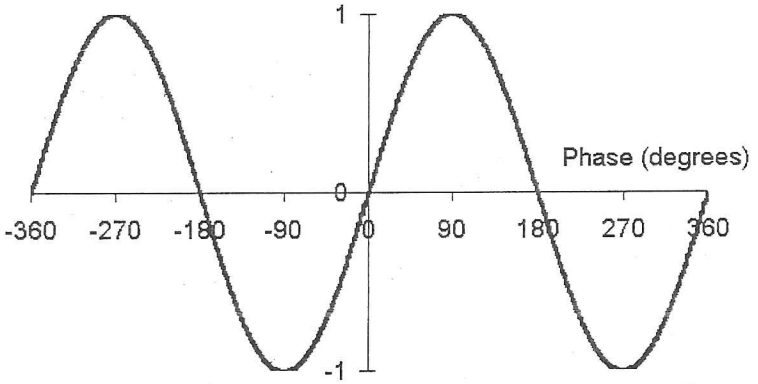
Function



Not a Function

Obtaining Information from a Graph:

Given:



Find: A. What are f(0), f(3π/2), and f(3π)? $f(0)=0$ $f(\frac{3\pi}{2})=-1$ $f(3\pi)=0$ (270° 540°)
B. What is the domain? All real numbers or $-2\pi \le x \le 2\pi$
C. What is the range? $-1 \le f(x) \le 1$
D. List the intercepts. $x = \pi K$ where K is an integer or $\{-2\pi, -\pi, 0, \pi, 2\pi\}$
E. How often does y=2 intersect the graph? Never
F. For what values does f(x) = -4? None
G. For what values is f(x) > 0? $2K\pi < x < \pi + 2K\pi$ or $\{-2\pi < x < -\pi$ and $0 < x < \pi\}$

Obtaining Information about the Graph of a Function:

Given: $f(x) = \frac{x}{x+2}$.

Find: A. Is the point (1, .5) on the graph?

B. If x = -1, what is f(x)? What point is on the graph of f?

C. If f(x) = 2, what is x? What ~~pint~~ point is on the graph of f?

A.) $\frac{1}{1+2} = \frac{1}{3}$ NO

B.) $f(-1) = \frac{-1}{-1+2} = \frac{-1}{1}$ $f(-1) = -1$ $(-1, -1)$

C.) $2 = \frac{x}{x+2}$ $2(x+2) = x$ $2x + 4 = x$ $4 = -x$ $-4 = x$ $(-4, 2)$

Ex. The average cost of manufacturing (C) of x computers per day is given by the function

$$C(x) = .56x^2 - 34.39x + 1212.57 + \frac{20{,}000}{x}$$

Determine the average cost of:

a. 30 computers a day 1351.54
b. 40 computers a day 1232.97
c. 50 computers a day 1293.07
d. Graph the function C = C(x), 0 < x < 80. Xscl = 10 Ymin = 0 Ymax = 5000 Yscl = 100
e. Create a table. Which value of x minimizes the average cost?

x	C(x)
39	1235.9
40	1233
41	1231.7
42	1232.2
43	1238.1

(41, 1231.7)

x = 41

HW: pg 7-10 #1-15, 18, 21, 23, 25, 26, 28, 31, 33, 36-38